A lens for linear angular mapping

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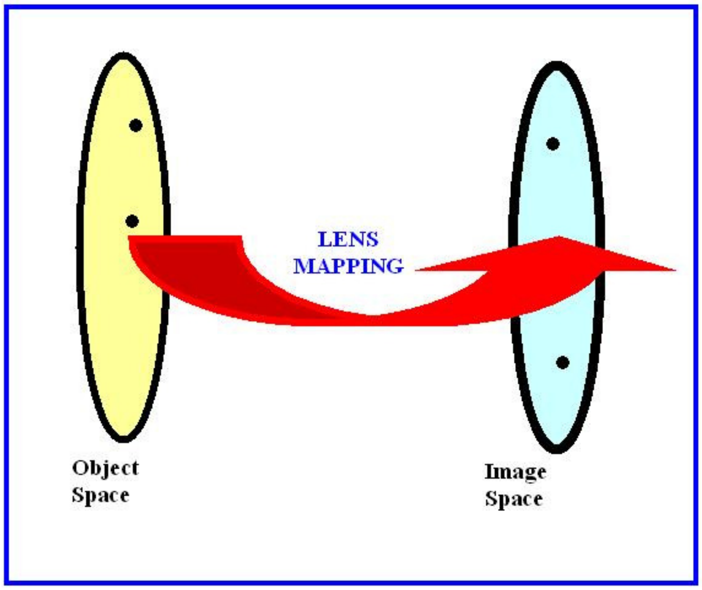


Abstract: The article suggests means to design a lens for angular measurement

About the author: Mr. Sharon serves as the CTO of CSTM Ltd, a Jerusalem based engineering company that specializes in undertaking engineering projects that combine optics, mechanics and electronics, Mr. Sharon holds an M.Sc. degree in applied physics from the Hebrew university (1992) and a B.Sc degree in Electro-Optics from the Jerusalem College of Technology (1984)

Introduction

An imaging lens is device that maps the object space onto the image space. For the mostly desired case of scaled image forming, the mapping is linear with the location:
every point p(x,y) in the object, is transformed to a point P(X,Y) in the image so that $X=\alpha x$ and $Y=\alpha y$
Is there a mapping that is linear with the view angle and not with the location? or regarding the plane of incidence, is there a lens that can transform a viewing angle Θ of the object to a translation Y in the image so that $Y=\alpha\Theta$?
The following analysis shows that there is.

Selected geometry

Assume an optimal PCX design in which light enters from the flat side (air to dielectric) and is focused by the convex side. Thus most of the distortion is governed by the refraction at the front.

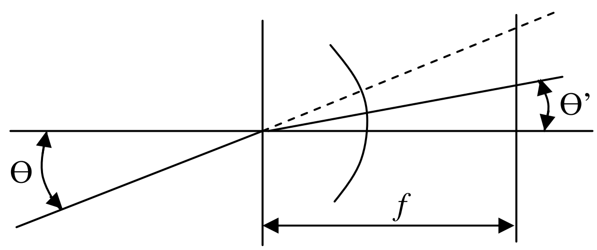


Fig.1: Selected PCX geometry

According to Snell's law: $Sin(\Theta) = nSin(\Theta')$ (i)

Differentiating (i) we get:

$$Cos(\Theta) = nCos(\Theta')\frac{d\Theta'}{d\Theta} \quad \text{or} \quad \frac{d\Theta'}{d\Theta} = \frac{Cos(\Theta)}{nCos(\Theta')} \qquad \text{(ii)}$$

Manipulating (i) we get:

$$Cos(\Theta') = [1 - \left(\frac{Sin(\Theta)}{n}\right)^2]^{1/2} \qquad \text{(iii)}$$

If imaging is obtained on a plane, the distance to the image point Y is given by:

Y= f tan(Θ')

Therefore: $\frac{dY}{d\Theta'} = \frac{f}{Cos^2(\Theta')}$

$$\frac{dY}{d\Theta} = \frac{dY}{d\Theta'}\frac{d\Theta'}{d\Theta} = \frac{f}{Cos^2(\Theta')}\frac{Cos(\Theta)}{nCos(\Theta')} = \frac{f}{n}\frac{Cos(\Theta)}{Cos^3(\Theta')} \qquad \text{(iv)}$$

Substituting (iii) into (iv) we obtain:

$$\frac{dY}{d\Theta} = \frac{f}{n}\frac{Cos(\Theta)}{\left[1-\left(\frac{Sin(\Theta)}{n}\right)^2\right]^{3/2}} \qquad \text{(v)}$$

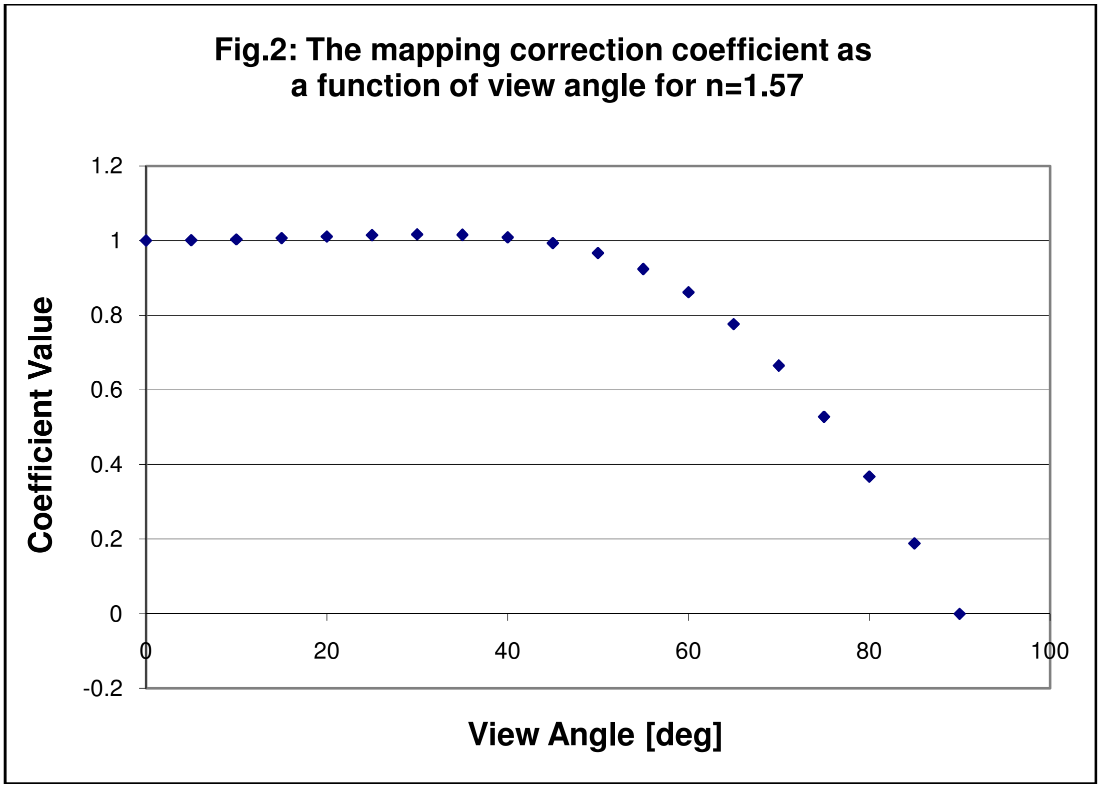
When $\Theta = 0$, $\frac{dY}{d\Theta} = \frac{f}{n}$

therefore an optimal system should give $\frac{dY}{d\Theta} = \frac{f}{n}$ for every angle Θ

This requires $\frac{Cos(\Theta)}{\left[1-\left(\frac{Sin(\Theta)}{n}\right)^2\right]^{3/2}}$ =1.

If we chart the mapping correction coefficient R(*n*,Θ) = $\frac{Cos(\Theta)}{\left[1-\left(\frac{Sin(\Theta)}{n}\right)^2\right]^{3/2}}$

as a function of n, we obtain a minimum close to 1 at n=1.57



Conclusions

A PCX with n=1.57 (for example Polycarbonate or Schott BAK1) will have a mapping close to linear with regard to the viewing angle, up to a view angle of about 45^{o}. Note that best results will be obtained when the central thickness of the lens is close to its radius of curvature. Such a lens may be found useful in any device in which the viewing angle is more important than the scaled image, such as a triangulation system. Using a lens that generates directly the desired mapping may prove advantageous in cases where calculation power is limited, and also cost-wise when a conventional imaging lens is an overkill.